DESIGN OF NONINTERACTING FLIGHT CONTROL SYSTEMS IN THE PRESENCE OF LARGE PARAMETER VARIATIONS

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ABSTRACT

This report summarizes the progress achieved to data under NASA Grant NSG 1213. In Section I, some observations on the perfect noninteraction for a discrete parameter set is given. Section II investigates the sensitivity of a decoupled flight control system with respect to system parameter variations. In Section III, a brief description of a set of computer programs developed for this project is given. The program listings will be furnished upon request.

I. Perfect noninteraction for a discrete parameter set

When parameter variations are defined over a discrete set of parameter values, it may be possible to compute a fixed control law which yields decoupled behavior for all admissible parameter values. This possibility was pursued as a first step to the decoupling problem since when a solution exists, it may be obtained by some fairly simple computations.

It has been shown by Gilbert [1] that the class of all feedback control laws of the form,

$$u = Fx + Gv,$$

which decouple the system,

$$\dot{x} = Ax + Bu, \quad y = Cx,$$

must have the following structure for F and G,

$$F = -D^{-1}A^{*} + \sum_{j=1}^{m} \sum_{k=1}^{k_i} (\sigma_{jk} - \pi_{jk}) J_k^j,$$

$$G = \sum_{j=1}^{m} \lambda_j G_j,$$

where the matrices A^{*},D,J_k^j and G, and the scalers π_{jk} are determined from the given matrices A,B and C, while σ_{jk} and λ_j are free design parameters. In addition σ_{jk} determine the poles of the closed-loop decoupled system, while λ_j determine the scalar gain factor of each diagonal element in the closed-loop transfer matrix. For each system parameter set A_i, B_i, there is a corresponding F_i, G_i with free parameter σ_{jk}^i and λ_j^i. The idea is then to seek values of σ_{jk}^i and λ_j^i such that

$$F_1 = F_2 = F_3 = \ldots\ldots ,$$

$$G_1 = G_2 = G_3 = \ldots\ldots .$$

This yields a system of linear equations in the free parameters. If a solution to these linear equations exists, then a fixed control law exists which decouples the system for all possible values of A, B and C. Computer programs are being developed for the generation of the appropriate linear equations and their solution. If no solution exists, perfect decoupling

is not possible over the given parameter set and one must proceed to a more complex design algorithm, such as the guaranteed-cost procedure described in the original proposal. The requirement of stability of the closed-loop system places further constraints on the existence of a solution. Finally the requirement of invariant closed-loop poles may be imposed by requiring that the same values of σ_{jk}^{i} and λ_{j}^{i} be used in each F_i and G_i. However the places even more constraints on the existence of solutions.

An abstract geometric approach to the problem of perfect decoupling over a discrete parameter set has recently been developed by Asher and Mulholland in [2]. However the approach presented here is more direct and easier to compute.

II. Sensitivity of decoupled system

The following example is taken from Pope [3], and involves the linearized longitudinal equations for a landing maneuver (STOL aircraft with a trim speed of 60 knots and a $7\frac{1}{2}$ degree angle of attack). Let x denote the system state with components,

x_1 = forward speed

x_2 = pitch angle

x_3 = pitch rate

x_4 = vertical velocity

and let u denote the control vector with components,

u_1 = elevator deflection

u_2 = thrust change.

Then the system dynamics are given by (see page 23 of [3])

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{1}$$

where

$$A = \begin{bmatrix} -.032 & -32.2 & 0 & .133 \\ 0 & 0 & 1 & 0 \\ .00137 & 0 & -.743 & -.0014 \\ -.02 & 4.2 & 96.5 & -.3 \end{bmatrix} \tag{2}$$

$$B = \begin{bmatrix} 0 & .65E\text{-}3 \\ 0 & 0 \\ -.989 & -.7E\text{-}5 \\ 3.0 & -.87E\text{-}3 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

In [2] a state feedback control law of the form $u = Fx + Gv$ is determined which decouples pitch rate and forward speed. The decoupling values of matrices F and G are computed from computer programs developed by Gilbert and Pivichny [4]. For the nominal parameter values given in (2) and (3), the following values are computed for F and G, see page 40 of [3]*,

$$F = \begin{bmatrix} .01193 & .6605 & .8665 & .3E\text{-}4 \\ -1.4892E3 & 4.9538E4 & 0 & -204.615 \end{bmatrix} \tag{4}$$

$$G = \begin{bmatrix} -.088 & -.0229 \\ 0 & 3.2308E3 \end{bmatrix} \tag{5}$$

A computer program was developed for the computation of the closed-loop transfer matrix $H(s)$, with elements $h_{ij}(s)$. The computer program was used to verify that the matrices given by (4) and (5) did indeed decouple the nominal system. The theory was verified with diagonal elements,

$$h_{11}(s) = \frac{0.087s}{s^2 + 1.6s + 1} \quad , \qquad h_{22}(s) = \frac{2.1}{s + 1}$$

as predicted in [3]. Off diagonal elements, while not identically zero, were smaller than the diagonal elements by a factor of 1E-5. The closed-loop characteristic polynomial is $(s^2 + 1.6s + 1)(s + 1)(s + .12199)$.

However contrary to the claims in [3], the solution appears to be very sensitive to parameter variations. For example, if the system parameter values are rounded off to two significant figures and the same values of F and G are used, the transfer element $h_{21}(s)$, which couples elevator deflection to forward speed, is no longer negligibly small. In particular, for the parameter variations corresponding to the above round-off modification of system parameters,

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* The values of F and G given on page 40 are evidently in error. Correct values are given here.

$$h_{21}(s) = \frac{-.0397(s^2 + 70.15s + 19.75)}{s^4 + 1.761s^3 + 1.098s^2 - .0973s - .1654}$$

The coupling between thrust and pitch rate remained essentially zero ($h_{12}(s)$ was smaller than remaining elements by a factor of 1E-5). The closed-loop system is unstable, with a pole at s = 0.33.

If the system parameters are modified to reflect a new trim speed of 55 knots, the A and B matrices become* (page 88 of [3]),

$$A = \begin{bmatrix} -.02912 & -32.2 & 0 & .12103 \\ 0 & 0 & 1 & 0 \\ .00125 & 0 & -.676 & -.00127 \\ -.0182 & 4.2 & 89.829 & -.273 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & .55E-3 \\ 0 & 0 \\ -.831 & -.6E-5 \\ 2.52 & -.73E-3 \end{bmatrix}$$

This corresponds to parameter variations of approximately 10%. If the nominal values of F and G are used for this new flight condition, decoupling between elevator deflection and forward speed is again destroyed. In this case $h_{21}(s)$ becomes,

$$h_{21}(s) = \frac{-.0397(s^2 + 70.15s + 19.75)}{s^4 + 1.761s^3 + 1.098s^2 - 0.0973s - 0.1654}$$

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* There is apparently a sign error in the elements a_{41} and b_{42} listed on page 88.

As in the previous case, parameter variations result in an unstable closed-loop system, In this case the unstable closed-loop pole is located at $s = 0.28$. This numerical example will be used to test the design procedure developed in this study.

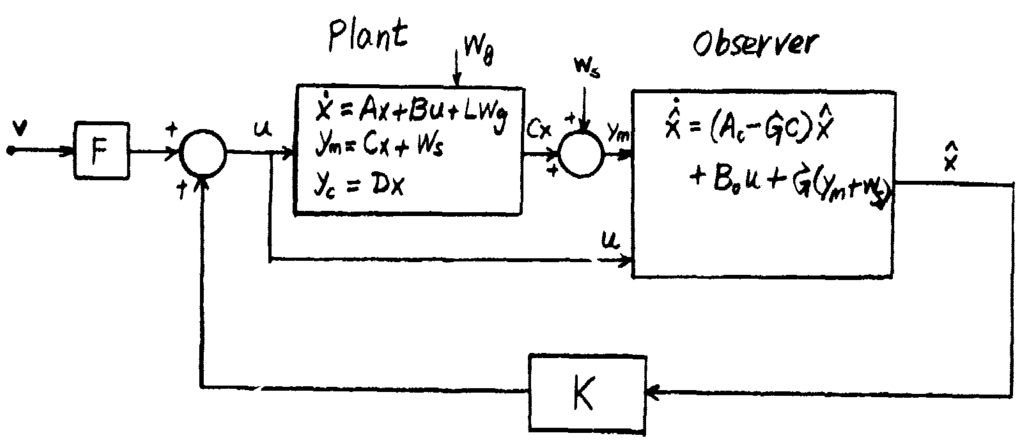
III. Computer programs

In this section, we describe the set of computer programs developed for this research project. This set of computer programs falls into two catagories (i) a digital simulation package and (ii) numerical optimization programs. All these programs are written in standard Fortran IV language. Hence they can be run on almost any computer installations. Some of these programs could be very time consumming when applied to high-order systems. It is planned to run most of them on Hewlett-Packard 2100 minicomputer which is owned by the University of Colorado at Colorado Springs. With proper arrangement of the operating system of the minicomputer, these program can be running in an interactive mode.

A brief description of these progrms is given below.

(i) Digital simulation package

This program is based on the Runge-Kutta fourth-order algorithm. The input data to the program is the parameter of the plant, observer,noises feedback and feedforward control laws, command input v, initial states x_0, $\hat{x}_0$, initial and final time t_0 and t_1. The over-all system configuration is shown below. The output of this program is the time history of the system outputs, which can be either printed in the graph form or stored in an array to be used in conjunction with the optimization programs described later.



(ii) Optimization programs

Two programs are included here, Box Complex Algorithm and Rosenbrock Hill Algorithm. Both algorithms find the maximum or minimum of a multivariable, nonlinear function subject to nonlinear inequality constraints:

$$\text{Optimize} \qquad F(X_1, X_2, \ldots, X_N)$$

$$\text{Subject to} \qquad G_k \le X_k \le H_k\,, \quad k = 1, 2, \ldots, M$$

The implicit variables $x_{N+1}, \ldots, X_M$ are dependent functions of the explicit independent variables, $X_1, X_2, \ldots, X_N$. The upper and lower constraints H_k and G_k are either constants or functions of the independent variables.

The Box Complex Algorithm is a sequential search technique which has proven effective in solving problems with nonlinear objective functions subject to nonlinear inequality constraints. No derivatives are required. The procedure should tend to find the global maximum due to the fact that the initial set of points are randomly scattered throughout the feasible region. The Rosenbrock Hill Algorithm proceeds per the unconstrained Rosenbrock procedure until convergence is reached or a boundary zone in the vicinity of the constraints is entered. Both of these two programs are taken from [5], which contains detailed information.

REFERENCE

[1] E. G. Gilbert, "The decoupling of multivariable systems by state feedback", SIAM J. Control, vol. 7, No. 1, February 1969, pp.50-61.

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[4] E. G. Gilbert and J. R. Pivichny,"A computer program for the synthesis of decoupled multivariable feedback systems", Proc. 1969 JACC.

[5] J. L. Kuester and J. H. Mize, "Optimization techniques with Fortran", McGraw-Hill Book Company, 1973.

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